High Dimensional Expanders

Luis Kumanduri

MIT

Definition

Let X be a d-dimensional simplicial complex. X is an ϵ -topological expander if for every continuous $F : X \to \mathbb{R}^d$, there is a point $p \in \mathbb{R}^d$ so that $F^{-1}(p)$ meets an ϵ fraction of the d-dimensional faces of X.

Theorem

(Gromov) If X has large cosystoles, satisfies a co-isoperimetric inequality and is sparse, then X is a topological expander.

Question

Can we develop better tests for expansion? In particular, what topological/geometric properties does expansion imply?

Question

Can we algorithmically estimate the expansion constant for a given complex? Somewhat relatedly, can we improve the bounds on the constant in Gromov's theorem?